higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1100(E)(J29)T
AUGUST 2011

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

29 July (X-Paper)
09:00 – 12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.

2. Answer ALL five questions in full. Show ALL the calculations and intermediary steps. Simplify where possible.

3. ALL the graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied. Values of intercepts with the system of axes and the turning point(s) MUST be shown on the graph.

4. ALL final answers must be accurately approximated to THREE decimal places.

5. Questions may be answered in any order but subsections of questions must NOT be separated.

6. A formula sheet is attached to this question paper. You are NOT compelled to use the formulae and the list is NOT necessarily complete.

7. Write neatly and legibly.

QUESTION 1

1.1 Given:

$$P=(Q+1)^{\frac{x}{t}}$$

Make t the subject of the formula. (4)

1.2 Sketch the graph of $y=\cot x;\ -90°\le x\le 90°$ (3)

1.3 1.3.1 Sketch the graph of the inverse of $y=\sqrt{16-x^2}$. (3)

1.3.2 What is the domain of the graph of the inverse of $y=\sqrt{16-x^2}$? (2)

1.4 Sketch the graph of $y=x^2-4x-5$. (3)

1.5 The perimeter of an enclosed field in the form of a right-angled triangle is 120 m. The length of the hypotenuse is 50 m. Determine the lengths of the other two sides of the triangle. (5)
[20]

QUESTION 2

2.1 Given:

$$2x+3y-z=4$$
$$3x+y+2z=13$$
$$x+2y-5z=-11$$

Solve for the value of x with the aid of Cramer's rule. (8)

2.2 Given:

$$(x+y)+j(x-y)=(2+j5)^2+j(2-j3)$$

Determine the values of x and y. (4)

2.3 Given:

$Z_1=3-j4$ and $Z_2=-5+j12$.

2.3.1 Determine the polar form of Z_1 and Z_2. (2)

2.3.2 Evaluate the following:

$$\frac{Z_1}{Z_2}$$

Leave the answer in $a + jb$ form. (4)

2.4 Simplify the following:

$3j^{16} - 2j^9$ (2)
[20]

QUESTION 3

3.1 Determine the value of sin 15° without the use of a calculator. Simplify the answer as far as possible. (4)

3.2 If $\cos A = \frac{12}{13}$ and A is an acute angle; show that $\sin 2A = \frac{120}{169}$ without the use of a calculator. (4)

3.3 Derive a formula for $\tan 2A$. Sketches may not be used. (3)

3.4 Given:

$\cos 2A = 2\cos^2 A - 1$

Derive a formula for $\cos\frac{A}{2}$. (4)

3.5 Show that:

$$\frac{\cos 2A}{\cos A \sin A} = \cot A - \tan A$$ (5)
[20]

QUESTION 4

4.1 Expand $(3 - 2x)^4$ to four terms only by the use of the binomial theorem. (4)

4.2 Given:

$y = \frac{1}{4}\sec 4x$

Differentiate with the aid of a function of a function rule. (4)

4.3 Differentiate with respect to x if:

$y = 2\log x - \tan 2x - 3\ell^{2x} + x^{-2}$ (4)

4.4 Given:

$y = 2x^3 - 5x^2 + 4x + 3$

Calculate the following:

4.4.1 Using differentiation, the co-ordinates of the maximum and minimum turning points

4.4.2 The co-ordinates of the point of inflection (8)
[20]

QUESTION 5

5.1 Integrate the following in terms of x:
$\int\left(4\sqrt[3]{x} - 3\cos ecx\cot x + 3\tan x - 2e^{7x} - 3^{6x} - 3t\right)dx$ (7)

5.2 Determine the following:
$\int_0^2 3e^x dx$ (3)

5.3 Simplify the following:

$\int\left(\frac{2}{\sqrt{x}} + 10^{-2x}\right)dx$ (3)

5.4 5.4.1 Sketch and indicate the area enclosed by the graph of $xy = 10$, the x-axis; $x = 1$ and $x = 5$. Indicate the representative strip to be used to calculate the enclosed area. (3)

5.4.2 Calculate by using integration the magnitude of the area indicated in QUESTION 5.4.1. (4)
[20]

TOTAL: 100

FORMULA SHEET

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ell n\, x = \log_e x$

$(r\,|\,\underline{\theta}\,)^n = r^n\,|\,\underline{n\theta}$ $\quad a + bj = c + dj \Leftrightarrow a = c$ and $b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \operatorname{cosec}^2 x$
$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
ka^x	$ka^x \ell na$
$k\ell nx$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$y = u(x) \cdot v(x)$

$$\Rightarrow \frac{dy}{dx} = u(x)v'(x) + u'(x)\,v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x} dx = a\ell n\, x + c$$

$$\int ka^x dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y\, dx$$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$\int \tan x\, dx = \ell n \sec x + c$

$\int \sec x\, dx = \ell n\,(\sec x + \tan x) + c$